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Advanced Cost Functions for Evaluation of Lateral Vehicle Dynamics

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Abstract The paper introduces the method of assessment of vehicle manoeuvres through a set of global cost functions. The corresponding cost functions can be derived for different domains like longitudinal and lateral dynamics, driving comfort and other. The procedures of the computation of the cost functions include in general: (1) Selection of vehicle dynamics parameters relevant to the domain; (2) Transformation the appointed parameter to the dimensionless form; (3) Definition of weighting factors for each of the appointed parameters with taking into account that the weighting factors can be variable depending on the type of the vehicle manoeuvre as well as on the driving conditions; (4) Calculation of the cost function for the selected domain; (5) Calculation of a global cost function in the case of the integrated assessment of the manoeuvre through several domains of the vehicle dynamics. The described procedures are discussed in the paper as applied to

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the domain of lateral vehicle dynamics. The parameters chosen for the calculation of the corresponding cost function are the lateral acceleration a_y, the yaw rate $d\psi/dt$, and the sideslip angle β. To transform these parameters to a dimensionless form, the procedure is proposed that uses the function of root mean square of deviations between reference and actual values for each variable. This procedure implements also an original method of definition of reference values for lateral acceleration a_y and yaw rate $d\psi/dt$. The method is based on the variation of understeer characteristic of the baseline vehicle with the aim to extend the linear region and to reduce the understeer gradient as well as to increase the maximum level of lateral acceleration. The validation of the developed methods and procedures is illustrated by way of model-in-the-loop simulation. The test programme covers several standard manoeuvres—steady-state circle, slalom and avoidance manoeuvre—performed for a simulator, medium-sized passenger car. The numerical values of the cost functions for each manoeuvre are introduced and analyzed. The further applications of the developed technique can be: (1) Assessment of vehicle dynamics based on criterions of performance and stability; (2) Optimization of vehicle dynamics control systems; (3) Choice of proper control strategies/tuning of control gains and resolution of critical control situations by simultaneous operation of several systems like ABS, TCS, TV/vehicle dynamics control.

Keywords Vehicle dynamics · Cost functions · Stability · Weighting factors · Simulation

Modern methods of vehicle dynamics control meet various complex challenges due to the fact that different systems with individual set of functions can be simultaneously involved in the control process: anti-lock braking, traction control, torque vectoring, direct yaw control, active suspension and so on. Resulting system fusion raises the issue about the development of an analytical tooling estimating the combined efficiency of the vehicle manoeuvres from viewpoint of longitudinal and lateral dynamics, ride comfort, driver control comfort, agility and other factors. Such a tooling can include a set of objective functions and variables interconnected through weighting factors, which depend on conditions of the driving manoeuvre, operational state of the vehicle and so on. For practical applications, a reasonable formulation of cost functions can be done in a dimensionless form.

An analysis of research literature points to lack of complex approaches to the evaluation of vehicle dynamics based on cost functions. However, several relevant studies should be mentioned in such a context. Milliken and Milliken [1] as well as Radt and Glemming [2] have proposed normalised, dimensionless description of tyre forces and moments, cumber and slip angles, and slip ratios. The mentioned works show that this approach is useful by assessment of combined lateral and longitudinal manoeuvres of the vehicle. Other studies have discussed more specified methods for the shaping the objective/cost functions in relation to the development of vehicle control systems. For example, the combined assessment of lateral and ride dynamics on the basis of frequency-dependent weighting index of the lateral acceleration has been given in [3]. The work [4] introduces objective functions of tractive performance as optimum slip and optimum input power for drive wheels that can be used in traction control systems.

The authors of the present article propose an extended flexible methodology that allows both individual and integrated evaluation of vehicle dynamics through diverse sets of cost functions. Next sections of the paper will introduce a relevant general approach, example of calculation of cost functions for lateral dynamics, and the case study illustrating the application of proposed method.

1 General Approach to the Calculation of Cost Functions

The efficiency of a vehicle manoeuvre can be evaluated for different domains of vehicle dynamics: longitudinal and lateral dynamics, ride comfort, driver comfort etc. Each domain has a set of inherent parameters. A parameter within a certain domain can be both independent and interrelated with other parameters. These statements are illustrated with Table 1.

The resulting diversity of parameters of vehicle dynamics implies many variations of possible cost functions as well as related computational methods. The authors of the present paper have proposed an approach that aims at the dimensionless interpretation of cost functions and their in-domain and inter-domain composition through a set of weighting factors. This approach is presented in Fig. 1 and can be explained as follows:

1. A set of parameters $N1 \ldots Nk$ is being chosen to shape the cost function of a certain domain N. The interpretation of parameters is preferred in a dimensionless form in the range 0–1, for example, as ratio of actual value and base value, or ratio of actual value and an appointed threshold.
2. An individual weighting factor has to be designated to each of parameters: $w_{N1} \ldots w_{Nk}$. At that the condition takes place:

$$\sum_{i=1}^{k} w_{Ni} = 1 \quad (1)$$

The magnitudes of weighting factors are not static and can be changed in accordance with the type of performed manoeuvre or actual driving conditions.

3. The corresponding cost function E_N for the domain N is calculated as

$$E_N = \sum_{i=1}^{k} N_i \cdot w_i \quad (2)$$

4. In accordance with postulates (1) and (2), the E_N–value from Eq. (2) yields a variable within the range 0–1.
5. In the case of a simultaneous evaluation of the vehicle manoeuvre in different domains, additional weighting factors can be designated to each domain-based

Table 1 Domains and parameters of vehicle dynamics – Example

Domain	Longitudinal dynamics	Lateral dynamics	Driver comfort
Parameters	Vehicle velocity	Lateral acceleration	Throttle pedal velocity
	Longitudinal acceleration	Yaw rate	Brake pedal velocity
	Wheel slip	Sideslip angle	Steering wheel velocity

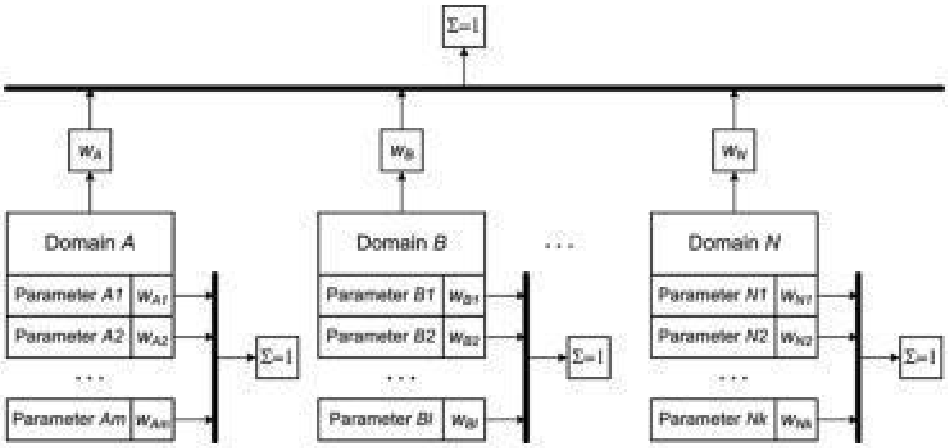


Fig. 1 Procedure of computing of cost functions

cost function (see factors w_A, w_B and w_N on Fig. 1). Their sum should be equal to 1, similar to postulate 2).

6. Implementation of positions (1)–(5) allows to compute an integrated, global cost function:

$$E_{global} = E_A \cdot w_A + E_B \cdot w_B + \ldots E_N \cdot w_N \tag{3}$$

The resulting cost function E_{global} lies also within the numerical interval 0–1.

Next parts of the paper will explain the proposed approach by the example of lateral vehicle dynamics.

2 Cost Functions of Lateral Vehicle Dynamics

2.1 Definition of Parameters of Lateral Vehicle Dynamics

The cost function, discussed in this section, is based on the vehicle parameters that (1) describe lateral vehicle dynamics and (2) can be measured by conventional on-board sensors, or sensors commonly adopted for vehicle testing: lateral acceleration a_y, side slip angle β, and yaw rate $d\psi/dt$. In such a case the basic formulation of the cost function can be proposed as

$$E_{lat} = w_{ay} \cdot f(\Delta a_y) + w_\beta \cdot f(\Delta\beta) + w_\psi \cdot f\left(\Delta\dot{\psi}\right), \tag{4}$$

where w_i are the weighting factors and

$$\Delta a_y = \left|a_y^{ref} - a_y^a\right|, \tag{5}$$

$$\Delta\beta = \left|\beta^{ref} - \beta^a\right|, \tag{6}$$

$$\Delta\dot{\psi} = \left|\dot{\psi}^{ref} - \dot{\psi}^a\right|. \tag{7}$$

Indexes *ref* and *a* in Eqs. (5)–(7) are relevant to the reference and actual values of the corresponding parameter.

The kind of function f in Eq. (4) depends on the purpose of the specific optimization tasks of vehicle dynamics. The analysis of reference literature shows that one of the most conventional variants can be based on the root mean square functions (RMS-functions). In particular, the methods using RMS error, relative RMS error, the mean relative error, and the maximum relative error are known. Within the framework of the discussed approach, the function of *root mean square* of deviations between reference and actual variables in Eqs. (5)–(7) has been chosen to assess vehicle dynamics during a certain manoeuvre :

$$E_{lat} = w_{ay} \cdot RMS\left(a_y^{ref}, a_y^a\right) + w_\beta \cdot RMS\left(\beta^{ref}, \beta^a\right) + w_\psi \cdot RMS\left(\dot{\psi}^{ref}, \dot{\psi}^a\right) \tag{8}$$

Equation (8) is written in a general form. Aiming at the deduction of dimensionless quantitative magnitudes of E_{lat}, the variants of relative RMS-deviations of actual and reference values can be proposed:

$$\begin{aligned} E_{lat} = w_{ay} \cdot \frac{f_{RMS}\left(\Delta a_y\right)}{\max\left(a_y^a\right) - \min\left(a_y^a\right)} + w_\beta \cdot \frac{f_{RMS}(\Delta\beta)}{\max(\beta^a) - \min(\beta^a)} + w_\psi \\ \cdot \frac{f_{RMS}\left(\Delta\dot{\psi}\right)}{\max\left(\dot{\psi}^a\right) - \min\left(\dot{\psi}^a\right)}, \end{aligned} \tag{9}$$

where

$$f_{RMS}(\Delta x) = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i^{ref} - x_i^a\right)^2}{n}}. \tag{10}$$

Table 2 Technical data of the vehicle

Parameter	Value/description
Total weight	2,080 kg
Maximum speed	201 kph
Acceleration 0–100 kph	8, 4 s
Front axle	McPherson suspension with lower triangular links and transverse torsion stabiliser
Rear axle	Multi-element suspension with a longitudinal and transverse links and transverse torsion stabiliser
Steering	Direct rack-and-pinion steering with electromechanical power steering
Tyres	215/60 R16
Dimensions	4,223 mm × 1,793 mm × 1,691 mm
Wheelbase	2,578 mm
Outer turning circle diameter	10, 32 m

In Eq. (10) the parameter n is the number of observed time points during the manoeuvre, x^{ref} and x^{a} in relation to Eq. (9) are the vectors of reference and baseline and lateral accelerations, yaw rates, or side slip angles given for the vehicle manoeuvre, for which the cost functions are being estimated. The *max*- and *min*-functions in Eq. (9) are maximum and minimum values of corresponding actual variable during the test. Following the discussed methodology, the ratio $f_{RMS}(x)/(max(x)-min(x))$—relative root mean square deviation—lies in the range [0, 1]. The value "0" should be considered as "the best case": the actual values of a variable coincide with the reference values. The value "1" should be considered as "the worst case".

The procedures of computing the reference characteristics for lateral acceleration, yaw rate and side slip angle are introduced in next sub-section.

2.2 Reference and Actual Values of Lateral Acceleration

The reference lateral acceleration is calculated as

$$a_y^{ref} = \begin{cases} a_y^*, & a_y^* \le a_{y\max} \\ a_{y\max}, \text{otherwise} \end{cases}. \tag{11}$$

The maximal lateral acceleration a_y* can be defined from the tyre friction ellipse as

$$a_{y\max} = 9,81 \cdot \mu_{y\max}, \tag{12}$$

where μ_{ymax} is the maximal lateral friction coefficient at given level of longitudinal acceleration/deceleration. The parameter a_y* in Eq. (11) identifies the reference lateral acceleration that does not exceed the friction limits and can be derived from

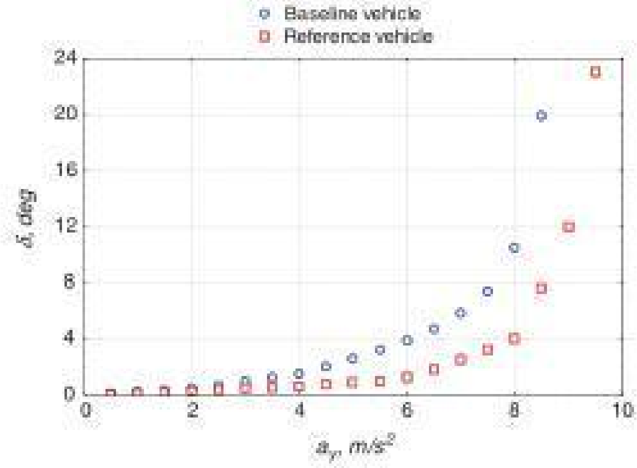


Fig. 2 Comparison of baseline and reference understeer characteristic at constant velocity

the look-up-table (LUT) given for the vehicle as a family of "Steering wheel angle—Lateral acceleration"—functions composed for different longitudinal accelerations. The procedure of shaping the reference characteristics is described below. The corresponding numerical examples are given for the vehicle with the technical data from Table 2.

Step 1. *Calculation of understeer characteristic of the baseline vehicle.* The understeer characteristics can be derived from results of various standard steady-state tests on vehicle dynamics. The regulations such as ISO 4138 or SAE J266 recommend different procedures: constant radius test, constant steer angle test, constant speed variable radius test, constant speed and variable steer angle test, or response gain test. An example of the understeer characteristic for a baseline vehicle is shown in Fig. 2. In the case under discussion the initial gradient of the curve is 0,375 °/m/s^2. The linear part of the "Steering wheel angle—Lateral acceleration"-dependence is valid until $a_y = 4.0$ m/s^2. The maximum level of lateral acceleration is $a_y = 8{,}25$ m/s^2.

Step 2. *Shaping the understeer characteristic of a reference vehicle for constant velocity conditions.* The reference understeer characteristic can be shaped after analysis of the baseline vehicle behaviour. It defines the target behaviour of reference vehicle dynamics that is characterized by:

- Less understeer gradient in term of $\delta(a_y)$;
- Extended linear part of the $\delta(a_y)$;
- Higher maximum level of lateral acceleration.

An example of reference understeer characteristics is introduced in Fig. 2. It was computed taking into account the mass-geometry parameters of the baseline vehicle as well as relevant tyre characteristics. The main reference characteristic parameters are: the initial gradient of the curve is 0.2 °/m/s^2; the extension of the linear part of the "Steering wheel angle—Lateral acceleration" characteristic-

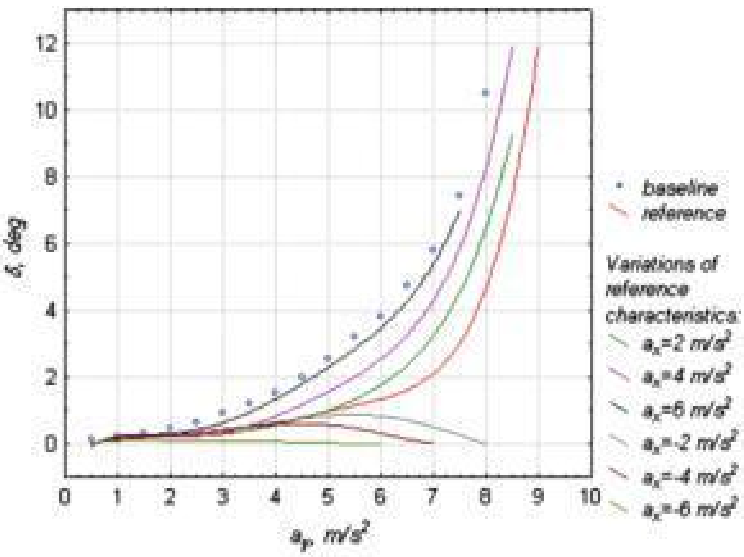


Fig. 3 A set of reference (a_y)-dependencies by variation of longitudinal acceleration

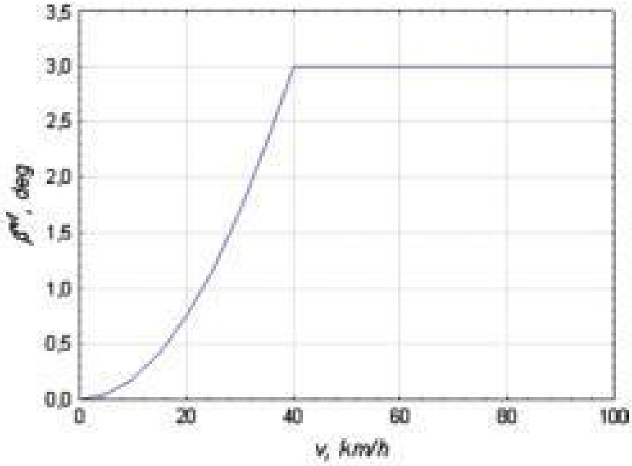
dependence (for the specific case the reference is linear until $a_y = 5.7$ m/s^2); the maximum reference level of lateral acceleration is $a_y = 9.5$ m/s^2.

Step 3. Variation of reference understeer characteristic. During this step, the behaviour of the reference understeer characteristic has to be defined for different levels of longitudinal acceleration a_x. Figure 3 proposes a tree of the reference $\delta(a_y)$ characteristics shaped for different a_x-levels. A set of displayed reference curves has common initial linear gradient. The end of the linear part depends on the given level of longitudinal acceleration. Such an approach is required to determine the variation of the understeer characteristic as a function of longitudinal acceleration. As it was evidenced by simulation and experimental results [5], the understeer characteristic can be influenced by the actual a_x-level with trends (1) to more understeering with the growth of a_x and (2) to oversteering by negative a_x-values. Hence, the reference $\delta(a_y)$-dependences are subject of variation. Referring to the tree of understeer characteristics from Fig. 3, the maximal a_y-value for each branch is limited by the corresponding a_x-level. Generally the shaped reference characteristics should have:

- Reduced understeer gradient during all a_y-range (while considering the traction of front wheel drive vehicle); in the case of rear wheel drive vehicle the reference characteristic can possess increased understeer;
- Reduced variation of the understeer characteristic subjected to a_x.

The value of actual lateral acceleration a_y^a is obtained either (1) with conventional lateral accelerometer being a component of vehicle dynamics control system or (2) from the vehicle simulator, or (3) from the vehicle model in the case of simulation [6]:

Fig. 4 The reference characteristic for side slip angle



$$a_y^a = \frac{1}{m_a}\left(\sum_{i=1}^{4} F_{xi}\sin(\delta_{vi}) - \sum_{i=1}^{4} F_{fi}\sin(\delta_{vi}) + \sum_{i=1}^{4} F_{yi}\cos(\delta_{vi}) - m_s h_s \ddot{\theta}\right) \quad (13)$$

where F_{xi} are the longitudinal tire forces, F_{yi} are the lateral tire forces, δ_{vi} are the steer angles, F_{fi} are the tyre rolling resistance forces, m_a is the vehicle mass, m_s is the vehicle sprung mass, h_s is the roll height, and is the vehicle roll angle.

2.3 Reference and Actual Values of Vehicle Side Slip Angle

The reference side slip angle is calculated as

$$\beta^{ref} = \begin{cases} \beta^*, & v \le v_{ss} \\ \beta_{\text{Max}\_ss}, \text{otherwise} \end{cases} \quad (14)$$

The β^*-value of side slip angle for steady-state conditions can be chosen as:

$$\beta^* = \beta_{\max\_ss}\frac{v^2}{v_{ss}^2} \quad (15)$$

where v is the actual absolute vehicle velocity, β_{max_ss} and v_{ss} are correspondingly the maximal side slip angle and absolute vehicle velocity given for the point where the influence of velocity on yaw rate becomes negligible. Equation (15) refers to the source [6], where $\beta_{max\_ss} = 3°$ and $v_{ss} = 40$ *m/s* have been recommended on the statistical basis from the experimental results for different types of vehicles. Therefore the reference characteristic for side slip angle is being proposed as shown on Fig. 4. It can be seen that a smooth increase of β^{ref} takes place in the range of velocities from 0 to 40 km/h. It is necessary while estimating the dynamic situations on surfaces with the low friction, where the vehicle stability can be critical already at small driving velocities with low side slip angles.

The value of actual vehicle side slip angle β^a can be computed as follows [7]:

$$\beta^a = \tan^{-1}\left(\frac{v_y}{v_x}\right) \tag{16}$$

where the actual longitudinal velocity is estimated as

$$v_x = \int\left(a_x + v_y \cdot \frac{d\psi}{dt}\right)dt \tag{17}$$

and the actual lateral velocity can be found as

$$v_y = \int\left(a_y - v_x \cdot \frac{d\psi}{dt}\right)dt \tag{18}$$

The values of vehicle accelerations a_x and a_y as well as yaw rate $d\psi/dt$ are obtained from corresponding vehicle sensors. Instead of the vehicle model, other ways for the estimation of β^a are (1) the use of vehicle simulator or (2) measurement technique. Similar tooling can be used also for the estimation of the individual contributions v_x and v_y.

Unlike the reference lateral acceleration, the reference side slip angle should be considered as a maximum allowed value during the manoeuvre performed. In this regard the comparison of β^a and β^{ref} takes place only in the case $\beta^a > \beta^{ref}$.

2.4 Reference and Actual Values of Yaw Rate

The calculation of reference yaw rate $(d\psi/dt)^{ref}$ is similar with the procedure for lateral acceleration that is described above. Generally

$$\dot{\psi}^{ref} = \begin{cases} \dot{\psi}^*, \ \dot{\psi}^* \leq \dot{\psi}_{\max} \\ \dot{\psi}_{\max}, \ \text{otherwise} \end{cases} \tag{19}$$

By analogy with the lateral acceleration, the parameter $(d\psi/dt)^*$ in Eq. (19) can be derived from the look-up-table given for the vehicle as a family of "Steering wheel angle—Yaw rate"-dependencies composed for variable longitudinal accelerations. Figure 5 introduces a tree of corresponding curves that were computed from the steady state circle test 42,5 m similar to the reference a_y-curves from Fig. 3. The maximum value of yaw rate [6] can be in addition controlled as

$$\dot{\psi}_{\max} = \frac{a_{y\max} - \dot{v}_x \cdot \sin\beta_{ref}}{v_x \cdot \cos\beta_{ref}} \tag{20}$$

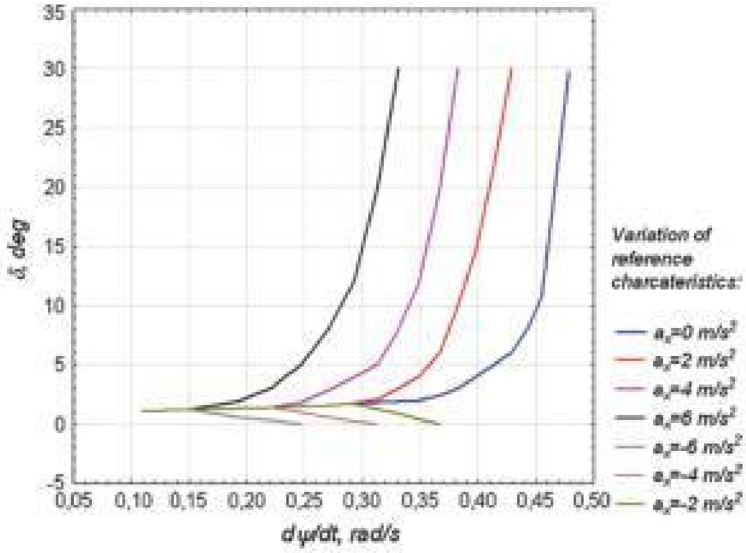


Fig. 5 A set of reference $\delta(d\psi/dt)$-dependencies by variation of longitudinal acceleration

Table 3 Examples of composition of weighting factors

Manoeuvre	w_{ay}	w_{β}	$w_{\dot{\psi}}$	Comment
Avoidance, dry road	0,3	0,1	0,6	Priority for yaw dynamics
Avoidance, ice	0,2	0,3	0,5	Higher priority for side slip due to low road friction
Track keeping on circle	0,6	0,05	0,35	Priority for lateral acceleration
Braking on mixed road	0,1	0,4	0,5	Higher priority for side slip angle dynamics
Slalom	0,4	0,2	0,4	Equal priority for yaw and lateral acceleration

The value of actual yaw rate $(d\psi/dt)^a$ can be obtained with conventional yaw rate sensor being a component of vehicle dynamics control system.

2.5 Choice of Weighting Factors

The next step is the choice of weighting factors for Eq. (8). It can be done for different driving situations, for instance:

- On-road straight-line manoeuvres;
- On-road manoeuvres with lateral dynamics;
- Off-road straight-line manoeuvres;
- Off-road manoeuvres with lateral dynamics.

It is appointed that the sum of three weighting factors w_{ay}, w_{β} and $w_{\dot{\psi}}$ is taken as 1. The magnitudes of weighting factors are selected depending on the kind of manoeuvre. The Table 3 gives examples of composition of weighting factors.

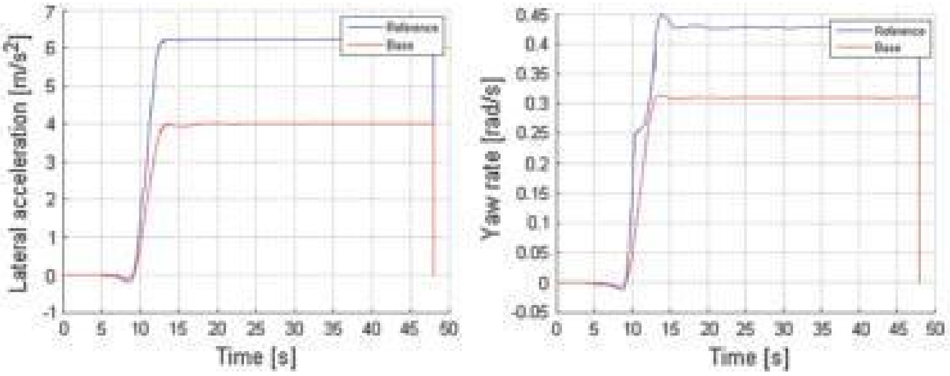


Fig. 6 Results of test of vehicle simulator, constant circle

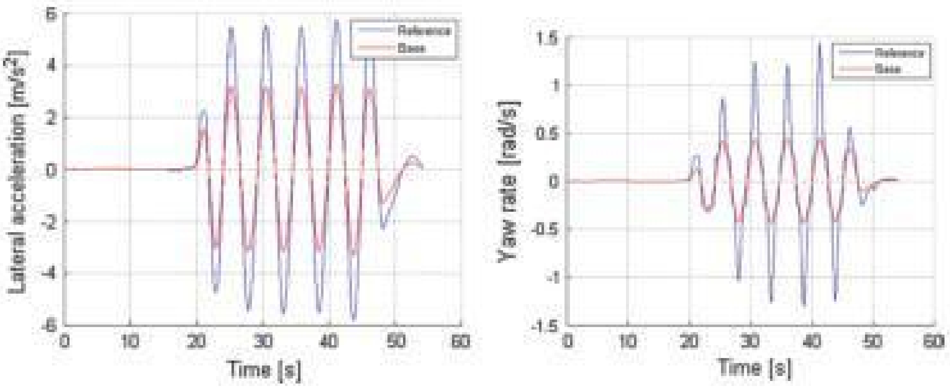


Fig. 7 Results of test of vehicle simulator, slalom

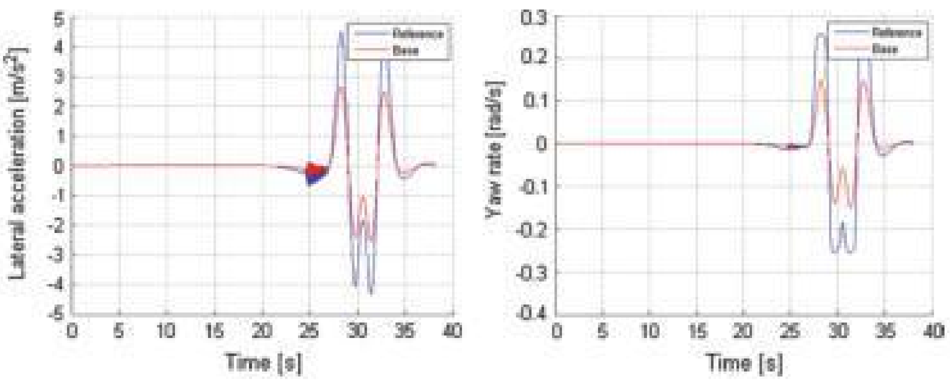


Fig. 8 Results of test of vehicle simulator, avoidance

It can be concluded from previous considerations that the magnitude of cost function E_{lat} from Eq. (9) for the certain manoeuvre will be always between 0 and

Table 4 Cost functions for results of tests of vehicle simulator (values given for the end of manoeuvre)

Test	**Constant circle**	**Slalom**	**Avoidance**
$f_{RMS}(\Delta a_y)$, m/s^2	1,8552	1,1069	0,5111
$f_{RMS}(\Delta\beta)$, deg	below threshold	1,1195	below threshold
$f_{RMS}(\Delta d\psi/dt)$, rad/s	0,0979	0,2353	0,0503
$max(\Delta a_y)$-$min(\Delta a_y)$, m/s^2	4,0064	6,5892	5,1872
$max(\Delta\beta)$-$min(\Delta\beta)$, deg	below threshold	4,3916	below threshold
$max(\Delta d\psi/dt)$- $min(\Delta d\psi/dt)$, rad/s	0,3103	0,8817	0,2990
w_{ay}	0,6	0,4	0,3
w_{β}	0,05	0,2	0,1
w_{ψ}	0,35	0,4	0,6
E_{lat}	**0,3883**	**0,3418**	**0,2590**

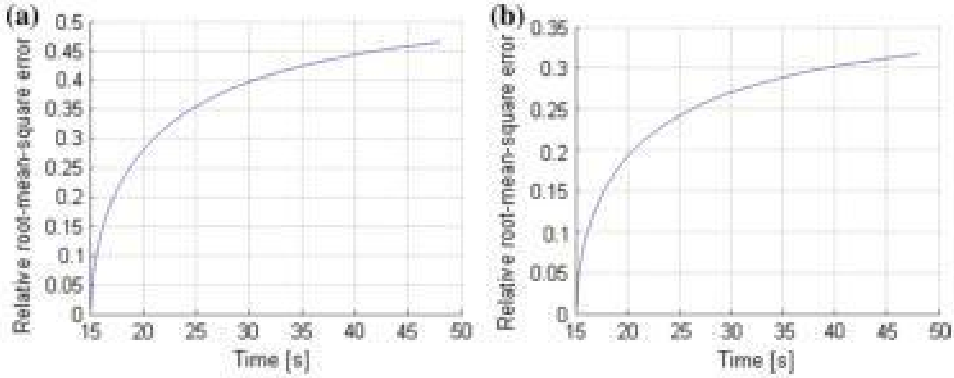


Fig. 9 The relative root mean square error functions from Eq. (9) Constant circle: **a** lateral acceleration; **b** yaw rate Critical part of the manoeuvre for calculation of cost functions - from 15 s

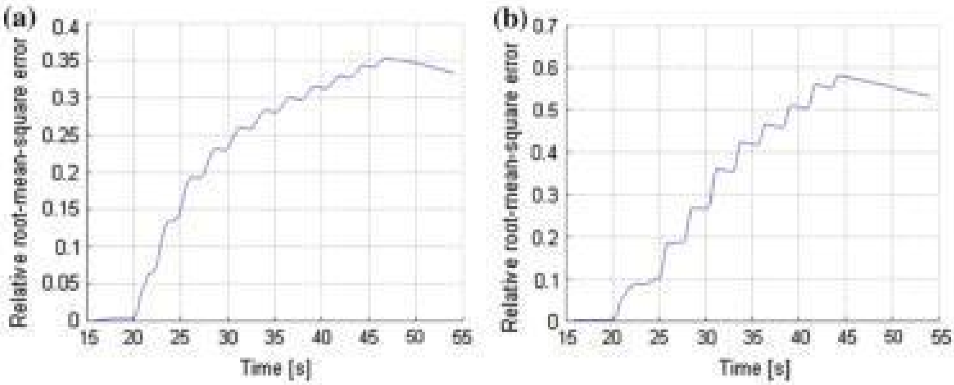


Fig. 10 The relative root mean square error functions from Eq. (9) Slalom: **a** lateral acceleration; **b** yaw rate Critical part of the manoeuvre for calculation of cost functions—15–55 s

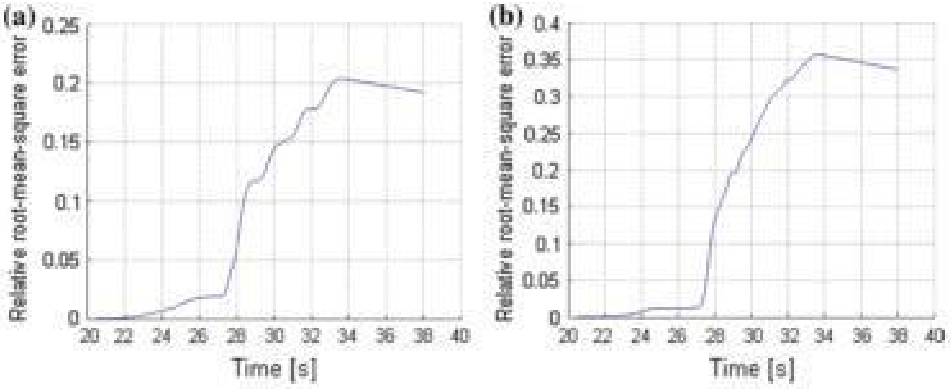


Fig. 11 The relative root mean square error functions from Eq. (9) Avoidance: **a** lateral acceleration; **b** yaw rate critical part of the manoeuvre for calculation of cost functions—20–38s

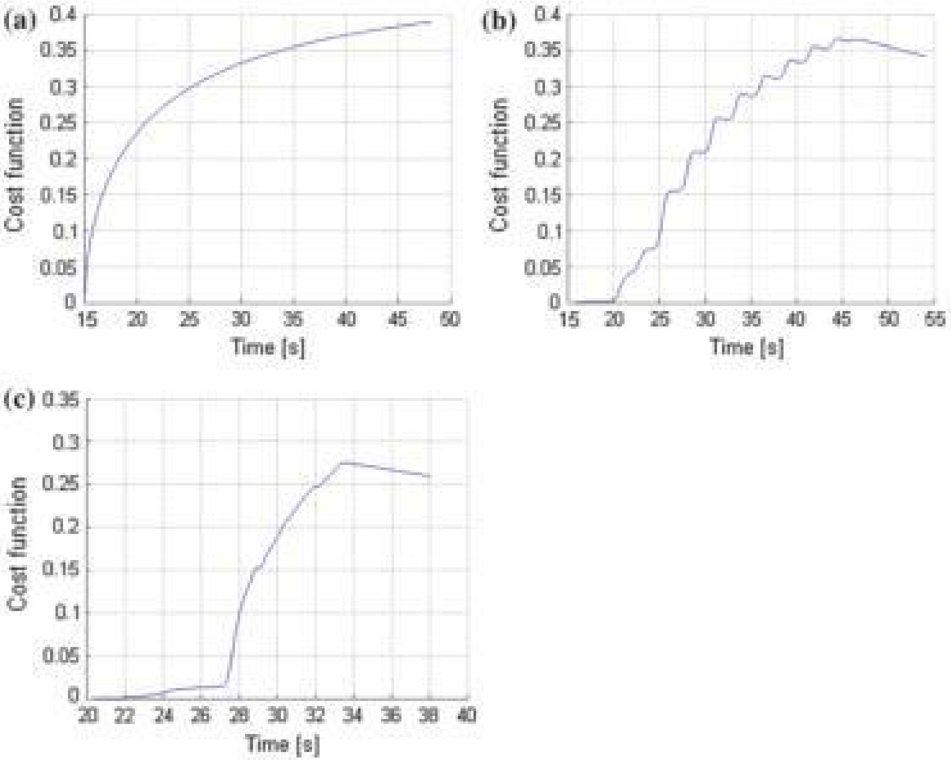


Fig. 12 Cost functions of lateral dynamics calculated in accordance with Eq. (9) **a** constant circle; **b** slalom; **c** avoidance

1. **The more E_{lat} tends to 0, the more efficiently the manoeuvre is performed from viewpoint of the lateral dynamics.**

3 Case Study for Application of Cost Functions

The case study was performed for the vehicle simulator created in the IPG CarMaker software for the car with the data from Table 2. Three following manoeuvres were simulated:

- Constant circle, radius 42,5 m;
- Slalom, 18 m;
- ISO avoidance manoeuvre.

The reference dependencies are similar with characteristics shown on Figs. 3, 4, 5. The comparison of actual and reference variables is shown on Figs. 6, 7, 8. The results of calculations of cost function are given in Table 4. The cost functions were computed for the critical parts of the manoeuvres. In addition Figs. 9, 10, 11 illustrates dynamics of the relative RMS-function for lateral acceleration and yaw rate, and Fig. 12 introduces dynamics of the cost function calculated in accordance with Eq. (9). It should be mentioned that the obtained results have indicated a proper evaluation of complexity of the performed manoeuvres.

The analysis of the case study allows to propose the following application areas of the cost functions:

- Assessment of vehicle dynamics based on criterions of performance and stability;
- Optimization of vehicle dynamics control systems;
- Choice of proper control strategies/tuning of control gains by simultaneous operation of several systems like ABS, TCS, and ESC.

4 Conclusions

The presented paper has introduced an approach to calculation of cost functions for the evaluation of lateral vehicle dynamics. The cost functions are based on the comparison of baseline and reference values of lateral acceleration, yaw rate and side slip angle. The following features can be especially mentioned in this context:

- The composition of reference variables can be obtained from the variation of understeer characteristics of the baseline vehicle. At that the reference vehicle should possess less understeering.
- The reference characteristics for lateral acceleration and yaw rate require the variation depending on the longitudinal acceleration.

- Composition of cost functions and weighting factors for singular components can be proposed in a dimensionless form in the range from 0 to 1.

The calculation of cost functions was illustrated with the case study for modelling of three different manoeuvres with the vehicle simulator.

The developed cost functions can be used for optimization of control strategy of automotive control systems and evaluation of vehicle dynamics.

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References

1. Milliken WF, Milliken DL (2002) Chassis design: principles and analysis. SAE International, p 676
2. Radt HS, Glemming DA (1993) Normalization of tire force and moment data. Tire Sci Technol 21(2):91–119
3. Peng H, Tomizuka M (1990) Vehicle lateral control for highway automation. In: Proceedings of american control conference, pp 788–794
4. Vantsevich VV (2007) Multi-wheel drive vehicle energy/fuel efficiency and traction performance: objective function analysis. J Terramech 44:239–253
5. Radt HS (1997) Variable dynamic testbed vehicle—analysis of handling performance with and without of rear steer, Milliken Research Associates Report
6. Kiencke U, Nielsen L (2005) Automotive control systems. Springer, Berlin-Heidelberg
7. Wong JY (2001) Theory of ground vehicles. Wiley, New York